3. (a) What is space group? (2)

(b) Derive the dispersion relation for a linear diatomic lattice. Sketch the dispersion relation indicating the two branches clearly in the graph. (4+2)

(c) What are meant by normal and Umklapp processes? (2)

4. (a) Prove the equivalence between vibrational mode in a solid and a harmonic oscillator. (6)

(b) What is geometrical structure factor? Show that the factor vanishes unless the number h, k and l are all even or all odd for f.c.c. lattice. (4)

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(Internal Assessment – 10)

2015

M.Sc.

1st Semester Examination

PHYSICS

PAPER – PGS-102 (Gr. – A + B)

Full Marks : 50

Time : 2 Hours

The figures in the right hand margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

(Gr. A – *Quantum Mechanics I*)

Answer Q1 and any one from Q2 and Q3.

1. Answer any five bits: 5 X 2 = 10

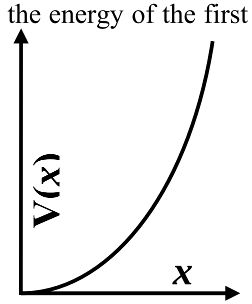
(i) A particle of mass m is confined in a two-dimensional square well potential of dimension a. The potential $V(x,y)$ is given by

$V(x,y) = 0$ for $-a < x < a$ and $-a < y < a$;

$= \infty$ elsewhere.

Find out the energy of the first excited state for this particle.

(ii) A particle is constrained to move in a truncated harmonic potential well ($x > 0$) as shown in the figure. Find out the energy of the first excited state of this particle with explanation.



(Turn Over)

(iii) The wavefunction of a particle is given by $\psi = \frac{\varphi_0}{\sqrt{2}} + \frac{i\,\varphi_1}{\sqrt{2}}$ where φ_0 and φ_1 are the normalized eigenfunctions with energies E_0 and E_1 corresponding to the ground state and the first excited state, respectively. Calculate the expectation value of the Hamiltonian in the state ψ.

(iv) Find the value of commutator $[L_z\,,\ cos\varphi]$ where φ is the azimuthal angle and $cos\varphi$ is the operator.

(v) Show that the norm of the state vector evolving from the Schrodinger Picture remains invariant with respect to time.

(vi) Deduce the parity of spherical harmonics $Y_{lm}\ (\theta\ ,\ \varphi)$.

(vii) For what values of the constant c will be the function $f(x) = A\, e^{-\alpha x}$ be an eigenfunction of the operator $\hat{Q} = \frac{d^2}{d^2x} + \frac{2}{x}\frac{d}{dx} + \frac{c}{x}$?

(vii) If $\hat{A}$ and $\hat{B}$ are Hermitian operators, prove that $(\hat{A}\hat{B} + \hat{B}\hat{A})$ is Hermitian but $(\hat{A}\hat{B} - \hat{B}\hat{A})$ is non-Hermitian.

2. (a) Considering the operator $a = \frac{i}{\sqrt{2m\hbar\omega}}(\hat{p} - im\hat{x})$ and its adjoint prove that $H|n\rangle = (n + 1/2)\hbar\omega|n\rangle$ for a linear harmonic oscillator where the symbols have their usual meanings. (4)

(b) Sketch the ground state wavefunction of hydrogen atom and hence, obtain the expectation value of r^2 in this state. (3)

(c) Discuss the equations of motion for both the wavefunction and operator in interaction picture.

3. (a) The wavefunction for a particle is represented as $\psi(\vec{r},t) = \sum_n a_n(t)\, u_n(\vec{r})$ where $H\, u_n(\vec{r}) = \ \varepsilon_n\, u_n(\vec{r})$. Show that $\langle\psi|H|\psi\rangle = \sum_n |a_n(t)|^2\, \varepsilon_n$. (3)

…Continued

(b) A three level quantum system has energy eigenvalues 0, 1, 2 MeV. If the probabilities for the system, at time t, to be in the first eigenstates are 49% and 36% respectively, write down the wavefunction $\psi(\vec{r},t)$ for the system. (2)

(c) Prove that the ground-state wavefunction of a linear harmonic oscillator is Gaussian. (2)

(d) A particle is confined to a box of length L with walls $x = 0$ and $x = L$. The particle is described by a wavefunction $\psi = N\sin(3\pi x/2L)\cos(\pi x/2L)$. Find the energy of the particle. (3)

(Gr. B – *Solid State Physics I*)

Answer Q1 and Q2 and any one from Q3 and Q4.

1. Answer any two bits: 2 X 2 = 4

(i) Show that five-fold rotational axis does not exist in a lattice system.

(ii) Find the packing fraction of the hcp structure.

(iii) The primitive lattice translation vectors of hexagonal space lattice may be taken as, $\vec{a_1} = \left(\frac{\sqrt{3}}{2}a\right)\hat{\imath} + \left(\frac{1}{2}a\right)\hat{\jmath};\ \vec{a_2} = -\left(\frac{\sqrt{3}}{2}a\right)\hat{\imath} + \left(\frac{1}{2}a\right)\hat{\jmath};\ \vec{a_3} = c\hat{k}$.

Show that the lattice is its own reciprocal, but with a rotation of axis.

2. Answer any two bits: 3 X 2 = 6

(i) What is Brillouin Zone? How can it be constructed using Bragg's diffraction condition?

(ii) Find the dispersion relation for mono-atomic lattice.

(iii) Find out the structure factor for the basis of diamond and prove that if all indices are even, the structure factor of the basis vanishes unless $h + k + l = 4n$, where n is an integer.

(Turn Over)