(b) Determine the normal mode frequency of the Lagrangian, given by $L = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}(\omega_1^2 x^2 + \omega_2^2 y^2) + \alpha\, xy$. (3)

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Internal Assessment-10

2018

M.Sc.

1st Semester Examination

PHYSICS

PAPER – PHS-101 (Gr. – A + B)

Full Marks : 50

Time : 2 Hours

Use separate answer scripts for Group A and Group B

(Methods of Mathematical Physics I - PHS 101A)

Answer Q1, Q2 and any one from Q3 and Q4

1. Answer any two bits: 2x2 = 4

(a) Prove that the function $f(z) = Arg\, Z,\ Z \in \mathbb{C} - \{0\}$ is nowhere differentiable, where $Arg\, Z$ denotes the principal value of argument of Z.

(b) If v_1 and v_2 be two orthonormal vectors in a Euclidean space, then prove that $||v_1 + v_2||^2 + ||v_1 - v_2||^2 = (||v_1|| + ||v_2||)^2$.

(c) If erf(x) and $\text{erf}_c(x)$ denote the error function and the complementary error function respectively, then prove that

$\lim_{\varepsilon \to 0+} \int_0^{\varepsilon} \text{erf}(x)\, dx = \lim_{\varepsilon \to 0+} \int_{\varepsilon}^{0} \text{erf}_c dx$.

(d) If A is an $n \times n$ matrix, shpw that $det\,(-A) = (-1)^n\, det\, A$.

2. Answer any two bits: 2x4 = 8

(a) Solve in series the Bessel differential equation:

$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$, when n is not an integer.

(b) Show that $\Gamma\left(n + \frac{1}{2}\right) = \frac{\sqrt{\pi}\,\Gamma(2n+1)}{2^{2n}\,\Gamma(n+1)}$.

(c) If $z \in \mathbb{C}$ and $|z| < \frac{\sqrt{5}-1}{2}$, then prove that $\frac{1}{1-z-z^2} = \sum_0^{\infty} a_n z^n$ where $\{a_n\}$ is the Fibonacci sequence 1,1,2,3,5,…

(Turn Over)

(d) Locate the singularities and evaluate the residues of the function $z^{-n}(e^z - 1)^{-1},\ z \neq 0$.

3. Use Gram-Schmidt process to obtain an orthonormal basis of the Euclidean space R^3 with standard inner product, generated by the linearly independent eigenvectors of the matrix $\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$. (8)

4. (a) Show that $\int_{-\infty}^{\infty} \frac{x\, sinx}{(x^2+a^2)} dx = \pi e^{-a}$. (5)

(b) How does the circles centered at the origin transform for $w(z) = \left(z - \frac{1}{z}\right),\ z \neq 0$? What happens when $|z| \to 1$? (3)

(*Classical Mechanics – PHS 101B*)

Answer Q1, Q2 and any one from Q3 and Q4

1. Answer any two bits: 2x2 = 4

(a) Prove that Poisson's Bracket of two dynamical variables remains invariant under infinitesimal canonical transformation.

(b) Show that the transformation defined by $q = \sqrt{2P}\ sinQ,\ p = \sqrt{2P}\ cosQ$ is canonical by using Poisson bracket conditions.

(c) What do you mean by action integral?

(d) Distinguish between point transformation and canonical transformation.

2. Answer any two bits: 2x4 = 8

(a) Outline Hamilton- Jacobi equation. What is Hamilton's principal function? Give its physical significance. (2+1+1)

(*Continued*)

(b) If G be a generating function depends only on (p_j, Q_j, t) then prove that $P_j = -\frac{\partial G}{\partial Q_j}$, $q_j = -\frac{\partial G}{\partial p_j}$; $\overline{H} = H + \frac{\partial G}{\partial t}$, the symbols have their usual meaning. What is identity transformation? (3+1)

(c) A particular mechanical system depending on two coordinates u and v has kinetic energy $T = v^2\dot{u}^2 + 2\dot{v}^2$ and potential energy $V = u^2 - v^2$. Write down the Lagrangian for the system and deduce its equation of motion. (Do not attempt to solve them).

(d) Show that the motion of a system during a small time 'dt' can be described by an infinitesimal contact transformation generated by the Hamiltonian of the system.

3. (a) Find the equation of motion of a hoop without slipping on an inclined plane and hence find its acceleration and frictional force of constant. (4)

(b) Prove that the transformation $p = 2\left(1 + q^{\frac{1}{2}}\, cosp\right) q^{\frac{1}{2}}\, sinp$,

$Q = \log\left(1 + q^{\frac{1}{2}}\ \mathrm{cos}p\right)$ is canonical and hence find the generating function. (4)

4. (a) If the Hamiltonian for a simple linear harmonic oscillator of mass m is given by $H = \frac{1}{2}\left(\frac{p^2}{m} + \mu q^2\right)$, (q, p) being position and momentum co-ordinates of the harmonic oscillator and $\mu = m\omega^2$, then find the corresponding Hamilton-Jacobi equation and determine the motion of the oscillator by using the Hamilton-Jacobi method. (3+2)

(Turn Over)